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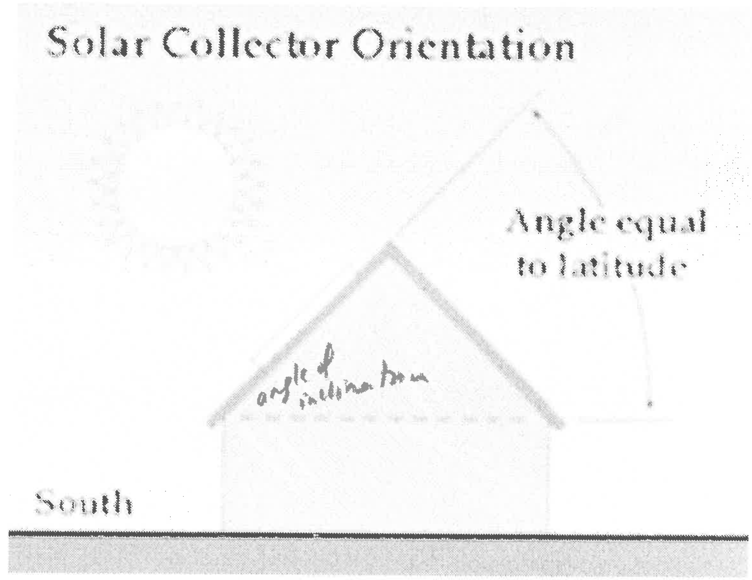
Ch. 2 Trigonometry Lessons and Practice

Introduction to Trigonometry

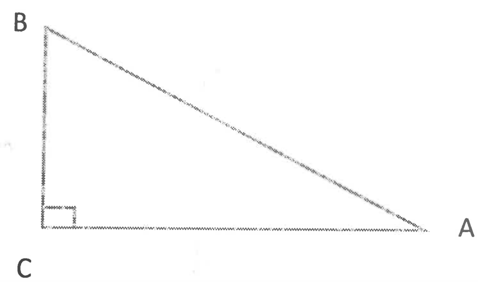
If you can never move the solar panel, place it at an angle equal to the degree of latitude.



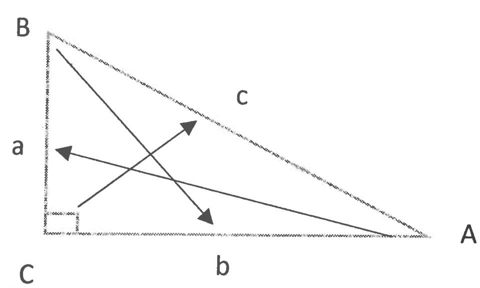
South-facing solar panels on a roof work best when the **angle of inclination** of the roof is approximately equal to the latitude of the house. The **angle of inclination** is the angle between the roof and the horizontal.

When an architect designs a house that will have solar panels on its roof, she has to determine the width and height of the roof so that the panels work efficiently.

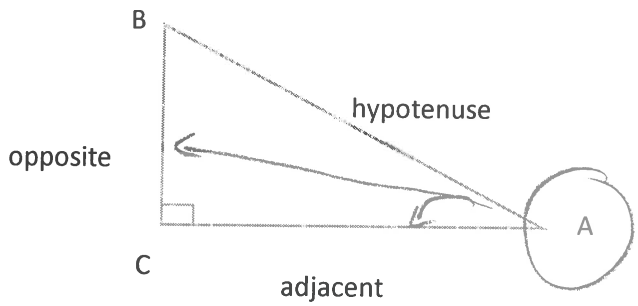
Is there an easier way to measure the angle than to use a protractor or some other measuring device? There is! It is called Trigonometry. We are going to work with only right triangles in this chapter. We name the sides of a right triangle in relation to one of its acute angles. What is an acute angle? (an angle less than 90 degrees)



But first, let's look at the letter names for the sides. Look at triangle ABC. Angles are indicated by capital letters and the sides across from them are named by lowercase letters.



We already know that the longest side in a right triangle is called the hypotenuse. The other two sides are called opposite and adjacent. But the **opposite and adjacent will change depending on which acute angle you are using as a reference point.** If we use angle A, the side across from it is the opposite, the side beside it (that isn't the hypotenuse) is the adjacent and the side across from the 90 degree angle is the hypotenuse.



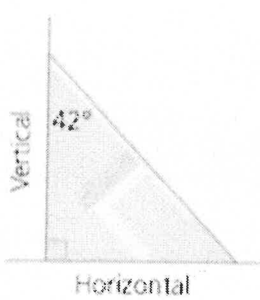
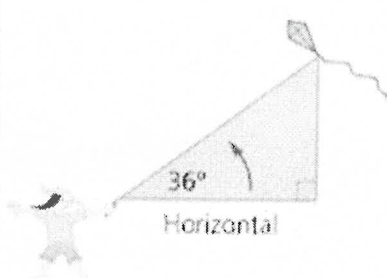
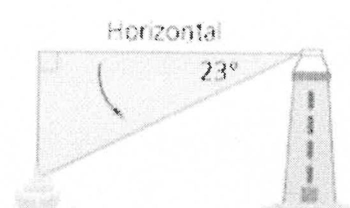
There are three primary trigonometric ratios. The ratio that we are going to learn in this section is the Tangent Ratio. It is the ratio of the opposite side over the adjacent side. We represent it as:

$$tanA = \frac{opposite}{adjacent} \quad \text{or} \quad \tan A = \frac{o}{a}$$

We often express it as a decimal that compares the lengths of the sides. For example, if tanA = 1.5, that means the opposite side is 1.5 times as big as the adjacent side.

We will learn about the other two ratios in a different lesson.

for future reference:

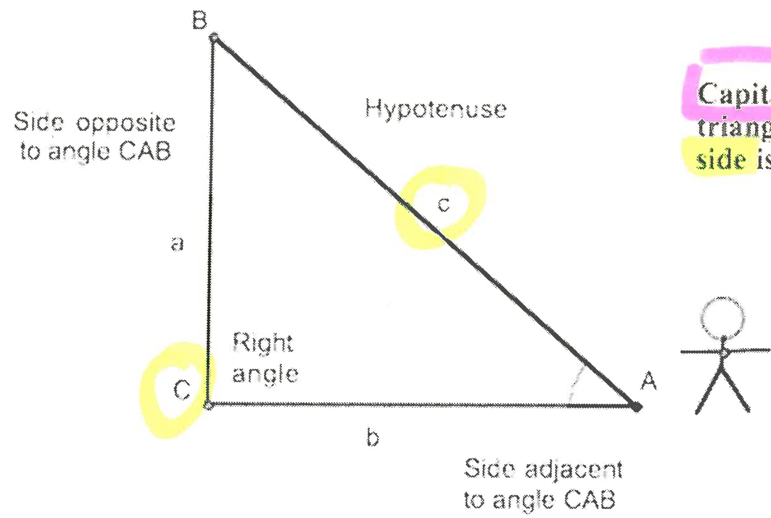
Angle of inclination	Angle of elevation	Angle of depression
Formed between the vertical or horizontal as defined by the situation. The angle of inclination with the vertical is 42°.	Formed by looking up from the horizontal The angle of elevation is 36°.	Formed by looking down from the horizontal. The angle of depression is 23°.
		

Sine, Cosine and Tangent - Three Functions, but same idea. We'll learn them separately and then put them all together.

- **Right Triangle**

Sine, Cosine and Tangent are the main functions used in Trigonometry and are based on a Right-Angled Triangle.

Before getting stuck into the functions, it helps to give a name to each side of a right triangle:



Capital letters name the vertices of the triangle. Lower case letters name the sides. A side is named after the angle opposite it.

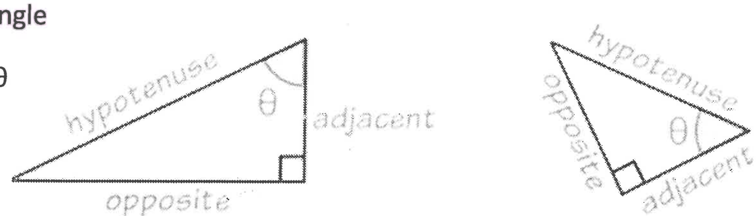
To make this easy picture yourself standing next to angle CAB. If you walk through the angle towards the right angle, you are walking on the *adjacent* side and the 'wall' you are approaching is the *opposite* side.

"Hypotenuse" is the long one and is **always** opposite (across from) the right angle. It doesn't change locations. (*Remember hypotenuse from Pythagoras' theorem*?)

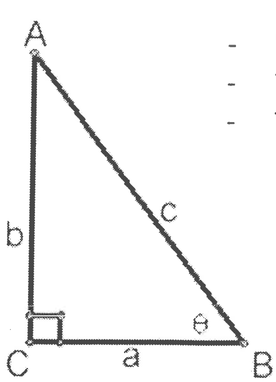
The adjacent and opposite **change** depending on which angle you're interested in (which we often label with the Greek letter, "theta", θ.)

"Opposite" is opposite to (across from) the angle

"Adjacent" is adjacent **(next to)** to the angle θ



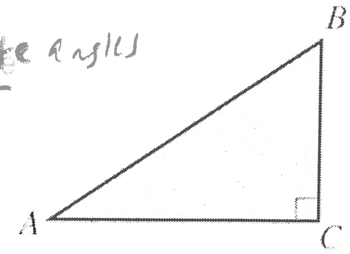
Try it: Identify:



- The side adjacent to θ ___a___
- The side opposite to θ ___b___
- The hypotenuse ___c___

A little bit of review.. finding missing angles, missing sides

- A useful and time-saving fact about right triangles is that the sum of the acute angles of any right triangle is 90°.

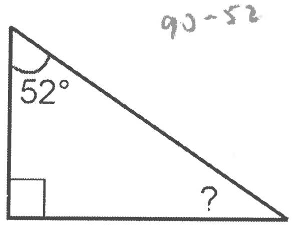


$A + B = 90°$

- We know that the sum of the interior angles of any triangle is 180°. A right triangle, by definition, contains a right angle, whose measure is 90°. That leaves 90° to be divided between the two acute angles.

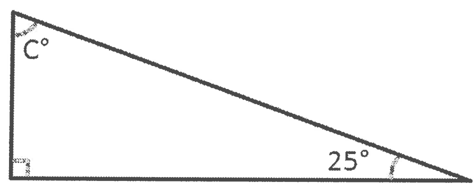
a) ? = 38°

90 − 52

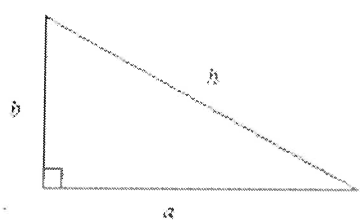


b) c = 65°

90 − 65



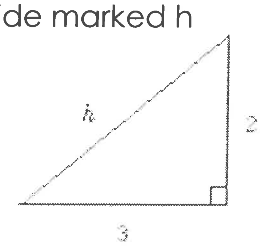
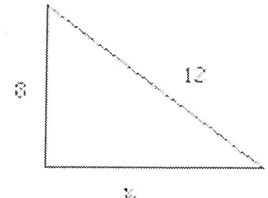
The **Theorem of Pythagoras** states that in a <u>right-angled triangle</u> the **square** of the length of the **hypotenuse** (the longest side) is equal to the **sum** of the **squares** of the lengths of the **two other sides** (the two shorter sides that form the right angle).



$$a^2 + b^2 = h^2$$

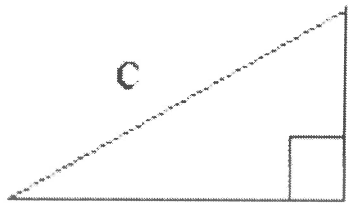
The hypotenuse, h, is always opposite the right angle.

The key to remember is that "**h**" is <u>always the hypotenuse</u> in this equation. The other 2 sides are transitive, so <u>either</u> side can be called "**a**" or "**b**".

Examples	Answers
Find the length of the side marked h 	$h^2 = 3^2 + 2^2$ $h^2 = 9 + 4$ $h^2 = 13$ $h = \sqrt{13}$
Find the length of the side marked x 	$12^2 = 8^2 + x^2$ $144 = 64 + x^2$ $144 - 64 = x^2$ $80 = x^2$ $\sqrt{80} = x$

Find the length of the third side of each right triangle, using Pythagoras. Show all work algebraically. Represent the length first as an entire radical and then write it as a decimal truncated (- chopped, not rounded) to 4 decimal places. When we are working on a problem and we need to round in the middle of a solution, we truncate to 4 decimal places. We only round (if needed) in the FINAL solution to the precision indicated.

3.



$8^2+4^2=c^2$

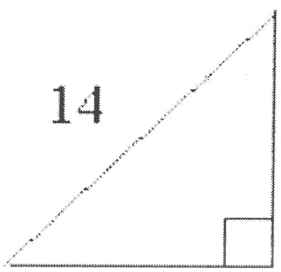
$64+16=c^2$

$\sqrt{80}=\sqrt{c^2}$

$\sqrt{80}=c$

$8.9442=c$

4.



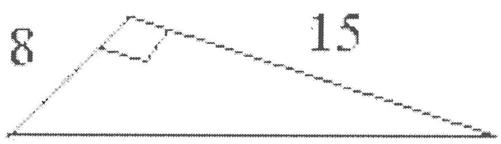
$a^2+11^2=14^2$

$a^2+121=196$

$\sqrt{a^2}=\sqrt{75}$

$a=\sqrt{75}=8.6602$

5.



$8^2+15^2=c^2$

$64+225=c^2$

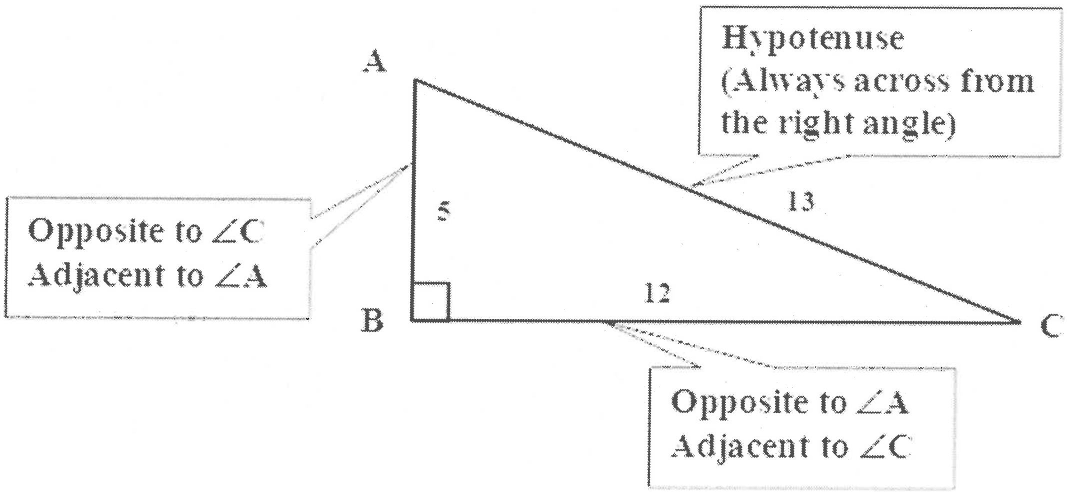
$289=c^2$

$17=c$

2.1 The Tangent Ratio

LESSON FOCUS: Develop the tangent ratio and relate it to the angle of inclination of a line segment.

Trigonometry is the study of the relationships among the **sides and angles** of triangles. One such relationship is the **tangent ratio**, which is an example of a trigonometric ratio.

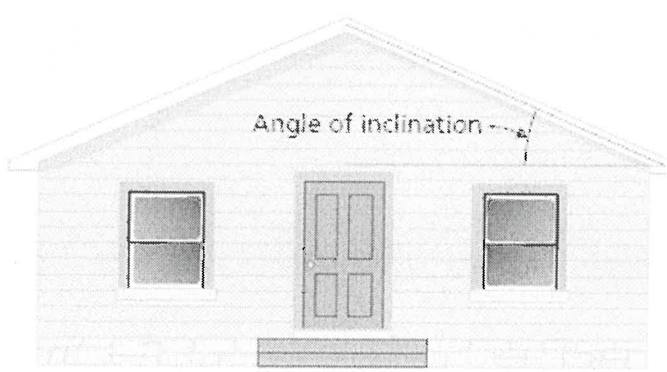


Definition: tangent ratio $= \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$

Short hand: $\boxed{\underbrace{\tan \overset{\text{Angle}}{\theta} = \frac{\text{opp}}{\text{adj}}}_{\text{Ratio}}} = \frac{o}{a}$

The **angle of inclination** of a line or line segment is the acute angle it makes with the horizontal.

(see also p. ii)



Knowing Your Calculator Make sure calculator says "deg"

Find the following to 3 decimal places.

a) tan 27° 0.510

0.5095 ↖ says change 5 to 1s.

b) tan 72°

3.078

Find ∠H to the nearest degree.

a) tan H = 4.332

∠H = 77°

b) tan H = 0.651

∠H = 33°

inverse tan
2nd tan

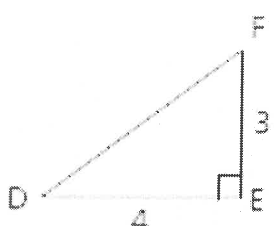
c) tan H = $\frac{3}{4}$ 3 ÷ 4

∠H = 37°

d) tan H = $\frac{9}{5}$ 9 ÷ 5

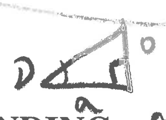
∠H = 61°

Example 1: Determining the Tangent Ratios for Angles
Determine tan D and tan F.



tan D = $\frac{\text{opp}}{\text{adj}}$

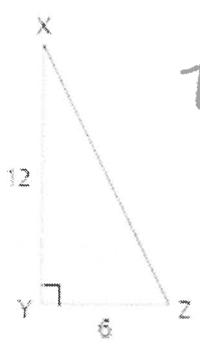
tan D = $\frac{3}{4}$



tan F = $\frac{o}{a}$

tan F = $\frac{4}{3}$

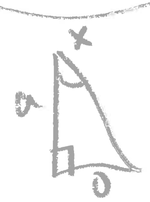
CHECK YOUR UNDERSTANDING
Determine tan X and tan Z. [Answer: tan X = 0.5; tan Z = 2]



tan X = $\frac{o}{a}$

tan X = $\frac{6}{12}$

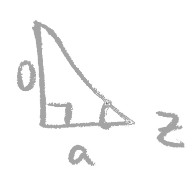
tan X = $\frac{1}{2}$



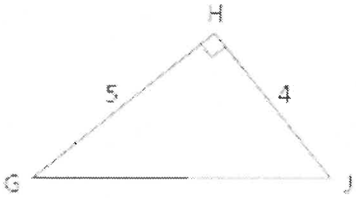
tan Z = $\frac{o}{a}$

tan Z = $\frac{12}{6}$

tan Z = 2



Example 2: Using the Tangent Ratio to Determine the Measure of an Angle
Determine the measures of ∠G and ∠J to the nearest tenth of a degree.

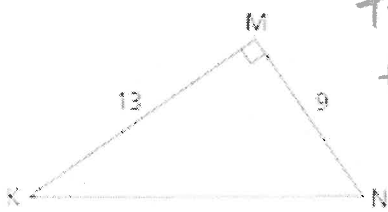


$\tan G = \frac{o}{a}$ $\tan J = \frac{o}{a}$ (put together 2 skills from p. 2 find ∠H and Example 1)

$\tan G = \frac{4}{5}$ $\tan J = \frac{5}{4}$

∠G = 38.7° ∠G = 51.3°

CHECK YOUR UNDERSTANDING
Determine the measures of ∠K and ∠N to the nearest tenth of a degree.
[Answer: ∠K□ 34.7°; ∠N□ 55.3°]



$\tan K = \frac{o}{h}$ $\tan N = \frac{o}{h}$

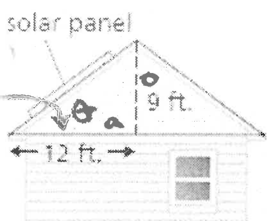
$\tan K = \frac{9}{13}$ $\tan N = \frac{13}{9}$

∠K = 34.7° ∠N = 55.3°

Example 3: Using the Tangent Ratio to Determine an Angle of Inclination
The latitude of Fort Smith, NWT, is approximately 60°. Determine whether this design for a solar panel is the best for Fort Smith. Justify your answer.

see p. 1 (see p. 1)

angle of inclination



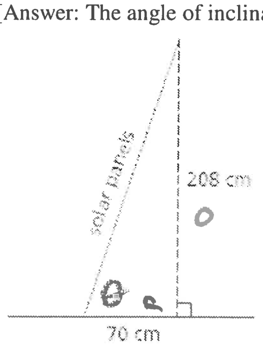
$\tan \theta = \frac{o}{a}$

$\tan \theta = \frac{9}{12}$

$\theta = 37°$

The angle of inclination is 37°, which is not equal to the latitude of Fort Smith. Not the best design for solar panel.

CHECK YOUR UNDERSTANDING
Clyde River on Baffin Island, Nunavut, has a latitude of approximately 70°. The diagram shows the side view of some solar panels. Determine whether this design for solar panels is the best for Clyde River. Justify your answer.
[Answer: The angle of inclination is approximately 71°. So, the design is the best.]



$\tan \theta = \frac{o}{a}$

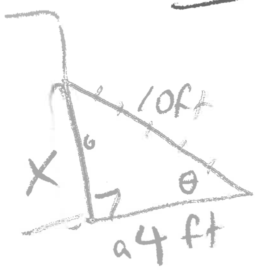
$\tan \theta = \frac{208}{70}$

$\theta = 71°$

This angle is close to the 70° latitude and so it is the best for Clyde River.

Example 4: Using the Tangent Ratio to Solve a Problem

A 10-ft. ladder leans against the side of a building with its base 4 ft. from the wall. What angle, to the nearest degree, does the ladder make with the ground?



$\tan\theta = \frac{o}{a}$

$\tan\theta =$ um... we don't know the opposite! Call it x.

Find opposite using Pythagoras.

$x^2 + 4^2 = 10^2$

$x^2 = 100 - 16$

$x^2 = 84$

$x = 9.1651$

Truncate 4 decimal places inside a solution.

Now ready! $\tan\theta = \frac{o}{a}$

$\tan\theta = \frac{9.1651}{4}$

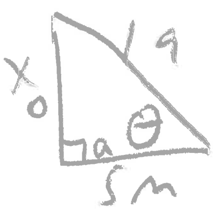
$\theta = 66°$

The ladder makes angle of 66° with ground.

CHECK YOUR UNDERSTANDING

A support cable is anchored to the ground 5 m from the base of a telephone pole. The cable is 19 m long. It is attached near the top of the pole. What angle, to the nearest degree, does the cable make with the ground?

[Answer: The angle is approximately 75°.]



$x^2 + 5^2 = 19^2$

$x^2 = 361 - 25$

$x^2 = 336$

$x = 18.3303$

$\tan\theta = \frac{18.3303}{5}$

$\tan\theta = 75°$

The cable make an angle of approximately 75° with the ground.

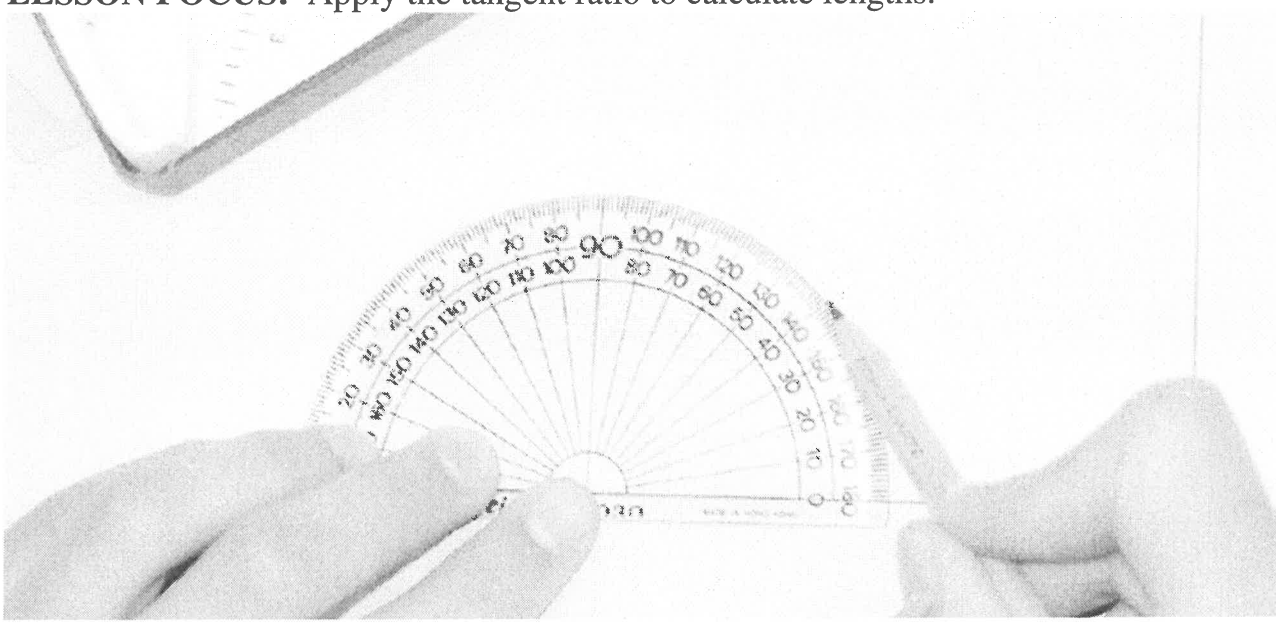
Homework: Page 75 **#3-5, 8, 10, 11, 13-16, 17**

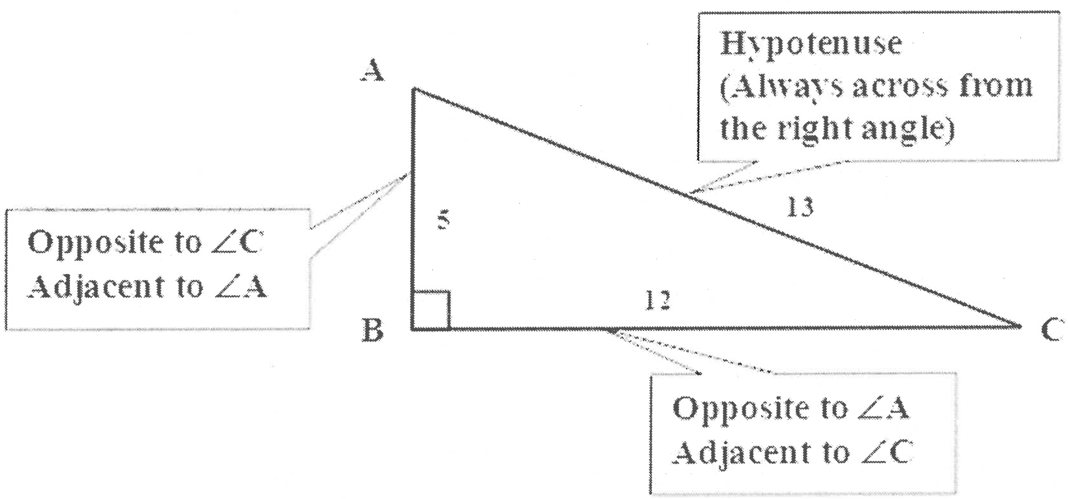
at least "a" of each

sum of ∠s of Δ = 180°

2.2 Using the Tangent Ratio to Calculate Lengths

LESSON FOCUS: Apply the tangent ratio to calculate lengths.



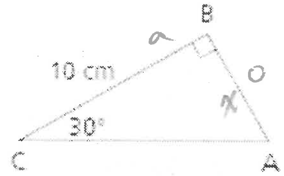


Definition: $\text{tangent ratio} = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$

Short hand: $\underbrace{\tan \overset{\text{Angle}}{\theta}} = \underbrace{\dfrac{\text{opp}}{\text{adj}}}$ $= \dfrac{o}{a}$

Ratio

Example 1: Determining the Length of a Side Opposite a Given Angle
Determine the length of AB to the nearest tenth of a centimetre.



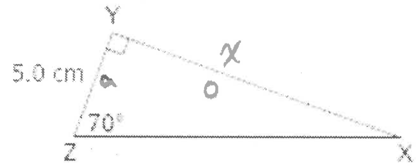
$\tan C = \frac{o}{a}$

$\tan 30 = \frac{x}{10}$ ← cross multiply

$10 \tan 30 = x$ ← do all at once on calculator (might have to press = after tan; might have to do tan 30 first, times 10)

$\boxed{5.8\text{cm} = x}$

CHECK YOUR UNDERSTANDING
Determine the length of XY to the nearest tenth of a centimetre. [Answer: XY□ 13.7 cm]



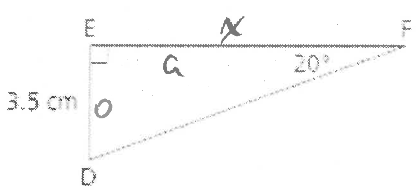
$\tan Z = \frac{o}{a}$

$\tan 70 = \frac{x}{5}$

$5 \tan 70 = x$

$\boxed{13.7\text{cm} = x}$

Example 2: Determining the Length of a Side Adjacent to a Given Angle
Determine the length of EF to the nearest tenth of a centimetre.



$\tan F = \frac{o}{a}$

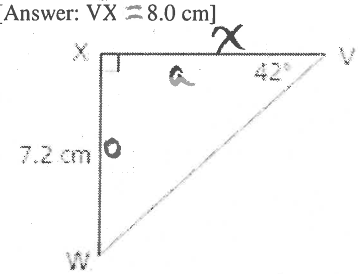
$\tan 20 = \frac{3.5}{x}$

$x \tan 20 = 3.5$ – cross multiply

$x = \frac{3.5}{\tan 20}$ – ÷ by tan 20

CHECK YOUR UNDERSTANDING
Determine the length of VX to the nearest tenth of a centimetre.
[Answer: VX ≃ 8.0 cm]

$\boxed{x = 9.6\text{cm}}$ $3.5 \div \tan 20$



$\tan V = \frac{o}{a}$

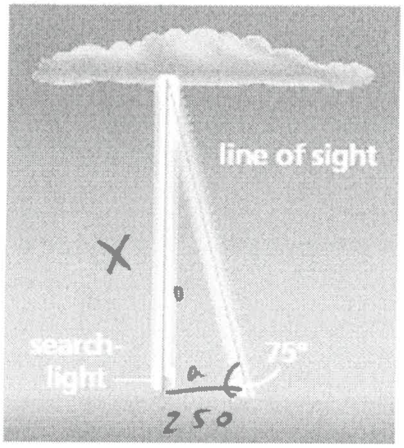
$\tan 42 = \frac{7.2}{x}$

$x \tan 42 = 7.2$

$x = \frac{7.2}{\tan 42}$ $x = 8.0\text{ cm}$

Example 3: Using Tangent to Solve an Indirect Measurement Problem
A searchlight beam shines vertically on a cloud. At a horizontal distance of 250 m from the searchlight, the angle between the ground and the line of sight to the cloud is 75°. Determine the height of the cloud to the nearest metre.

angle of elevation (see p. 11)



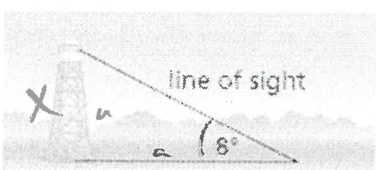
$\tan\theta = \frac{o}{a}$

$\tan 75 = \frac{x}{250}$

$250 \tan 75 = x$

$\boxed{x = 37\text{ m}}$ — height of cloud

CHECK YOUR UNDERSTANDING
At a horizontal distance of 200 m from the base of an observation tower, the angle between the ground and the line of sight to the top of the tower is 8°. How high is the tower to the nearest metre? The diagram is *not* drawn to scale.
[Answer: 28 m]



$\tan\theta = \frac{o}{a}$

$\tan 8 = \frac{x}{200}$

$200 \tan 8 = x$

$\boxed{28\text{ m} = x}$

tower height

Homework: Page 82 #3–8, 9a, 10, 11

3–5 at least